Resonance and Guitar Strings

Read from Lesson 5 of the Sound and Music chapter at The Physics Classroom:
http://www.physicsclassroom.com/Class/sound/u11l5a.html
http://www.physicsclassroom.com/Class/sound/u11l5b.html

MOP Connection: Sound and Music: sublevels 6 and 7

Review
1. Standing wave patterns consist of nodes and antinodes. The positions along a medium that appear to be stationary are known as __________. They are points of no displacement. The positions along a medium that are undergoing rapid motion between a maximum positive and maximum negative displacement are known as __________. They are the opposite of the points of no displacement.

2. Use the diagram below to compare the distance between two adjacent nodes on a standing wave pattern and the wavelength of a wave. Write a sentence comparing these two distances.

Resonance in Strings:
3. Draw the standing wave patterns for the first five harmonics and complete the equations.

<table>
<thead>
<tr>
<th>Harmonic #</th>
<th>Standing Wave Pattern</th>
<th>$\lambda \rightarrow L$</th>
<th>$L \rightarrow \lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>$L = \frac{1}{2} \lambda$</td>
<td>$\lambda = \frac{2}{1} L$</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>$L = \frac{2}{2} \lambda$</td>
<td>$\lambda = \frac{2}{2} L$</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>$L = \frac{3}{2} \lambda$</td>
<td>$\lambda = \frac{2}{3} L$</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>$L = \frac{4}{2} \lambda$</td>
<td>$\lambda = \frac{2}{4} L$</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>$L = \frac{5}{2} \lambda$</td>
<td>$\lambda = \frac{2}{5} L$</td>
</tr>
</tbody>
</table>
Sound and Music

4. Determine the wavelength of the ...

<table>
<thead>
<tr>
<th>a. ... wave in this 1.3-meter long string.</th>
<th>b. ... wave in this 85-cm long string.</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ L = 2\lambda ] [ 1.3 = 2\lambda ] [ \lambda = \frac{1.3}{2} ]</td>
<td>[ L = \frac{3}{2}\lambda ] [ 85 = \frac{3}{2}\lambda ] [ \lambda = \frac{85}{3} ]</td>
</tr>
<tr>
<td>c. ... first harmonic wave pattern for a 78.5-cm long guitar string.</td>
<td>d. ... fifth harmonic wave pattern for a 1.05-m long guitar string.</td>
</tr>
<tr>
<td>[ L = \frac{1}{2}\lambda ] [ 78.5 = \frac{1}{2}\lambda ] [ \lambda = 78.5 \times 2 ]</td>
<td>[ L = \frac{5}{2}\lambda ] [ 1.05 = \frac{5}{2}\lambda ] [ \lambda = \frac{1.05}{5} \times 2 ]</td>
</tr>
</tbody>
</table>

Use the wave equation and your standing wave patterns to solve the following problems. PSYW

5. A guitar string with a length of 80.0 cm is plucked. The speed of a wave in the string is 400. m/sec. Calculate the frequency of the first harmonic. PSYW

\[
 f_1 = \frac{v}{2L} = \frac{400}{2 \left( 0.8 \right)} = 250 \text{ Hz}
\]

6. Calculate the frequency of the second and third harmonic for the string in question #5. PSYW

\[
 f_2 = \frac{2v}{2L} = \frac{2 \left( 400 \right)}{2 \left( 0.8 \right)} = 500 \text{ Hz}
\]

\[
 f_3 = \frac{3v}{2L} = \frac{3 \left( 400 \right)}{2 \left( 0.8 \right)} = 750 \text{ Hz}
\]

7. A pitch of Middle D (first harmonic = 294 Hz) is sounded out by a vibrating guitar string. The length of the string is 70.0 cm. Calculate the speed of the standing wave in the guitar string. PSYW

\[
 f_1 = \frac{v}{2L} = 294 = \frac{v}{2 \left( 0.7 \right)}
\]

\[
 v = 412 \text{ m/s}
\]

8. A frequency of the first harmonic is 587 Hz (pitch of D5) is sounded out by a vibrating guitar string. The speed of the wave is 600. m/sec. Find the length of the string. PSYW

\[
 f_1 = \frac{v}{2L} = 587 = \frac{600}{2L}
\]

\[
 L = 0.51 \text{ m}
\]

9. A rope is vibrating in such a manner that three equal-length segments are found to be vibrating up and down with 321 complete cycles in 20.0 seconds. Waves travel at speeds of 26.4 m/s in the rope. What is the length of the rope? PSYW

\[
 f = \frac{321}{20} = 16.05 \text{ Hz}
\]

\[
 \lambda = \frac{2L}{3} \quad f = \frac{3v}{2L} = \frac{3 \left( 26.4 \right)}{2 \times 2.5} = 16.05
\]

\[
 L = 2.5 \text{ m}
\]
Resonance and Open-End Air Columns

Read from Lesson 5 of the Sound and Music chapter at The Physics Classroom:
http://www.physicsclassroom.com/Class/sound/u115a.html
http://www.physicsclassroom.com/Class/sound/u115c.html

MOP Connection: Sound and Music, sublevels 8 and 9

Review
1. Standing wave patterns consist of nodes and antinodes. The positions along a medium that appear to be stationary are known as __nodes__. They are points of no displacement. The positions along a medium that are undergoing rapid motion between a maximum positive and maximum negative displacement are known as __antinodes___. They are the opposite of the points of no displacement. Each consecutive node is separated from each other by \( \frac{1}{2} \lambda \).

2. Define fundamental frequency:
   - The first resonance frequency of an object
   - The natural frequency of the object

Resonance in Open-End Air Columns:
3. An open-end air column is a column of air (usually enclosed within a tube, pipe or other narrow cylinder) that is capable of being forced into vibrational resonance. Both ends of the column are open to the surrounding air. Air at the ends of the column is able to vibrate back and forth. Thus, these ends form vibrational __antinodes__ (nodes, antinodes).

4. Draw the standing wave patterns for the first five harmonics and complete the equations.

<table>
<thead>
<tr>
<th>Harmonic #</th>
<th>Standing Wave Pattern</th>
<th>( \lambda \rightarrow L )</th>
<th>( L \rightarrow \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><img src="image1" alt="Standing Wave Pattern" /></td>
<td>( L = \frac{1}{2} \lambda )</td>
<td>( \lambda = \frac{2}{L} )</td>
</tr>
<tr>
<td>2</td>
<td><img src="image2" alt="Standing Wave Pattern" /></td>
<td>( L = \frac{\lambda}{2} )</td>
<td>( \lambda = \frac{2}{L} )</td>
</tr>
<tr>
<td>3</td>
<td><img src="image3" alt="Standing Wave Pattern" /></td>
<td>( L = \frac{3}{2} \lambda )</td>
<td>( \lambda = \frac{2}{L} )</td>
</tr>
<tr>
<td>4</td>
<td><img src="image4" alt="Standing Wave Pattern" /></td>
<td>( L = \frac{4}{2} \lambda )</td>
<td>( \lambda = \frac{2}{L} )</td>
</tr>
<tr>
<td>5</td>
<td><img src="image5" alt="Standing Wave Pattern" /></td>
<td>( L = \frac{5}{2} \lambda )</td>
<td>( \lambda = \frac{2}{L} )</td>
</tr>
</tbody>
</table>

5. Determine the frequency of the ....
   a. ... third harmonic for an air column whose first harmonic frequency is 384 Hz. \( \frac{3(384)}{4} = 1152 \text{ Hz} \)
   b. ... first harmonic for an air column whose fourth harmonic frequency is 1296 Hz. \( \frac{1296 \text{ Hz}}{4} = 324 \text{ Hz} \)
   c. ... third harmonic for an air column whose fourth harmonic frequency is 528 Hz. \( \frac{528 \text{ Hz}}{4}, 3 = 396 \text{ Hz} \)
Sound and Music

6. Determine the wavelength of the...

<table>
<thead>
<tr>
<th>a. wave in this 63-cm long air column.</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ L = \frac{3}{2} \lambda ]</td>
</tr>
<tr>
<td>[ 63 \text{ cm} = \frac{3}{2} \lambda ]</td>
</tr>
<tr>
<td>[ \lambda = 42 \text{ cm} ]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>b. wave in this 85-cm long air column.</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ L = 2 \lambda ]</td>
</tr>
<tr>
<td>[ 85 = 2 \lambda ]</td>
</tr>
<tr>
<td>[ \lambda = 42.5 \text{ cm} ]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>c. first harmonic wave pattern for a 42.5-cm long air column.</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \frac{L}{\lambda} = \frac{1}{2} \lambda ]</td>
</tr>
<tr>
<td>[ 42.5 \text{ cm} = \frac{1}{2} \lambda ]</td>
</tr>
<tr>
<td>[ \lambda = 85 \text{ cm} ]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>d. fifth harmonic wave pattern for a 140-cm long air column.</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \frac{L}{\lambda} = \frac{1}{2} \lambda ]</td>
</tr>
<tr>
<td>[ 140 \text{ cm} = \frac{1}{2} \lambda ]</td>
</tr>
<tr>
<td>[ \lambda = 0.56 \text{ m} ]</td>
</tr>
</tbody>
</table>

Use the wave equation and your standing wave patterns to solve the following problems. PSYW

7. Stan Dinghwaives is playing his open end pipe. The frequency of the second harmonic is 882 Hz (a pitch of A♯). The speed of sound through the pipe is 345 m/sec. Find the frequency of the first harmonic and the length of the pipe. PSYW

\[
\begin{align*}
    f_2 &= 2f_1 \\
    882 &= 2f_1 \\
    f_1 &= 441 \text{ Hz} \\
    \lambda &= 0.782 \text{ m} \\
    L &= \frac{1}{2} \lambda \\
&= \frac{1}{2} (0.782) \\
&= 0.39 \text{ m}
\end{align*}
\]

8. A flute is played with a first harmonic of 196 Hz (a pitch of G3). The length of the open-end air column is 89.2 cm (quite a long flute). Find the speed of the wave resonating in the flute. PSYW

\[
\begin{align*}
    f_1 &= 196 \text{ Hz} \\
L &= \frac{1}{2} \lambda \\
\lambda &= 2L \\
\lambda &= 2 \times 89.2 \\
\lambda &= 178.4 \text{ cm} \\
\lambda &= 1.784 \text{ m} \\
\lambda &= \frac{v}{f} \\
v &= \lambda f \\
v &= 196 \times 1.784 \\
v &= 349.7 \text{ m/s}
\end{align*}
\]

9. Find the length of a flute which would resonate at 262 Hz on a day when the speed of sound in air is 345 m/s. PSYW

\[
\begin{align*}
    v &= \lambda f \\
345 &= 262 \lambda \\
\lambda &= 1.32 \text{ m} \\
L &= \frac{1}{2} \lambda \\
&= \frac{1}{2} (1.32) \\
&= 0.66 \text{ m}
\end{align*}
\]

10. Find the frequency of a 63.8-cm long open end air column that resonates as shown in the diagram at the right. The speed of sound in the air is 345 m/s.

\[
\begin{align*}
    L &= \frac{3}{2} \lambda \\
\lambda &= \frac{2}{3} L \\
\frac{f}{\lambda} &= \frac{3v}{2L} \\
\frac{f}{\lambda} &= \frac{3(345)}{2(63.8)} \\
&= 811 \text{ Hz}
\end{align*}
\]
Resonance and Closed-End Air Columns

Read from Lesson 5 of the Sound and Music chapter at The Physics Classroom:
http://www.physicsclassroom.com/Class/sound/u11l5a.html
http://www.physicsclassroom.com/Class/sound/u11l5d.html

MOP Connection: Sound and Music: sublevels 10 and 11

Review
1. Standing wave patterns consist of nodes and antinodes. The positions along a medium which appear to be stationary are known as _______ nodes _______. They are points of no displacement. The positions along a medium which are undergoing rapid motion between a maximum positive and maximum negative displacement are known as _______ antinodes _______. They are the opposite of the points of no displacement. Each consecutive node is separated from each other by _______ λ ______.

Closed

Resonance in Open-End Air Columns:
2. A closed-end air column is a column of air (usually enclosed within a tube, pipe or other narrow cylinder) which is capable of being forced into vibrational resonance. One end of the column is closed to the surrounding air and the other end is open to the surrounding air. Air at the open end of the column is able to vibrate back and forth; this end forms a vibrational _______ antinode _______ (node, antinode). Air at the closed end is NOT able to vibrate back and forth; this end forms a vibrational _______ node _______ (node, antinode).

3. Draw the standing wave patterns for the first five harmonics and complete the equations.

<table>
<thead>
<tr>
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<th>$L \rightarrow \lambda$</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td><img src="image1" alt="Standing Wave Pattern" /></td>
<td>$L = \frac{1}{4} \lambda$</td>
<td>$\lambda = 4L$</td>
</tr>
<tr>
<td>3</td>
<td><img src="image2" alt="Standing Wave Pattern" /></td>
<td>$L = \frac{3}{4} \lambda$</td>
<td>$\lambda = \frac{4}{3} L$</td>
</tr>
<tr>
<td>5</td>
<td><img src="image3" alt="Standing Wave Pattern" /></td>
<td>$L = \frac{5}{4} \lambda$</td>
<td>$\lambda = \frac{4}{5} L$</td>
</tr>
<tr>
<td>7</td>
<td><img src="image4" alt="Standing Wave Pattern" /></td>
<td>$L = \frac{7}{4} \lambda$</td>
<td>$\lambda = \frac{4}{7} L$</td>
</tr>
<tr>
<td>9</td>
<td><img src="image5" alt="Standing Wave Pattern" /></td>
<td>$L = \frac{9}{4} \lambda$</td>
<td>$\lambda = \frac{4}{9} L$</td>
</tr>
</tbody>
</table>

4. Determine the frequency of the ....
   a. third harmonic for an air column whose first harmonic frequency is 262 Hz. \( \frac{3(262)}{2} = 786 \text{ Hz} \)
   b. first harmonic for an air column whose fifth harmonic frequency is 1700 Hz. \( \frac{5(1700)}{2} = 3400 \text{ Hz} \)
   c. fifth harmonic for an air column whose third harmonic frequency is 984 Hz. \( \frac{5(984)}{2} = 1560 \text{ Hz} \)
   d. next highest frequency for an air column whose fundamental frequency is 210 Hz. \( \frac{3(210)}{2} = 315 \text{ Hz} \)

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Sound and Music

6. Determine the wavelength of the...
   a. ... wave in this 35-cm long air column.
      \[ L = \frac{5}{4} \lambda \]
      \[ 3.5 = \frac{5}{4} \lambda \]
      \[ \lambda = 2.8 \text{ cm} \]

   b. ... wave in this 56-cm long air column.
      \[ L = \frac{7}{4} \lambda \]
      \[ 56 = \frac{7}{4} \lambda \]
      \[ \lambda = 3.2 \text{ cm} \]

   c. ... first harmonic wave pattern for a 32-cm long air column (closed).
      \[ L = \frac{1}{4} \lambda \]
      \[ 32 = \frac{1}{4} \lambda \]
      \[ \lambda = 128 \text{ cm} \]

   d. ... fifth harmonic wave pattern for a 1.20-meter long air column (closed).
      \[ L = \frac{5}{4} \lambda \]
      \[ 1.2 = \frac{5}{4} \lambda \]
      \[ \lambda = 0.96 \text{ m} \]

7. The Test Tubes have a gig in the local park this weekend. The lead instrumentalist uses a test tube (closed end air column) with a 17.2 cm air column. The speed of sound in the test tube is 340 m/sec. Find the frequency of the first harmonic played by this instrument. PSYW
   \[ \frac{L}{4L} = \frac{\lambda}{4L} \]
   \[ f = \frac{\sqrt{\frac{340}{L}}}{4L} = 494 \text{ Hz} \]

8. A closed end organ pipe is used to produce a mixture of sounds. The third and fifth harmonics in the mixture have frequencies of 1100 Hz and 1833 Hz respectively. What is the frequency of the first harmonic played by the organ pipe? PSYW
   \[ f_3 = 3f_1 \]
   \[ f_5 = 5f_1 \]

9. Pipin' Pete and the Pop Bottles is playing at Shades next weekend. One of the pop bottles is capable of sounding out a first harmonic of 349.2 Hz. The speed of sound is 345 m/sec. Find the length of the air column. PSYW
   \[ f = \frac{\sqrt{\frac{345}{L}}}{4L} \]
   \[ 349.2 = \frac{3 \times 345}{4L} \]
   \[ L = 0.247 \text{ m} \]

10. The sound produced by blowing over the top of a partially filled soda pop bottle is the result of the closed-end air column inside of the bottle vibrating at its natural frequency. Keri Atune uses four bottles (labeled A, B, C and D) with varying amounts of water (and thus, air) in order to play a song. Express your understanding of closed-end resonance by filling in the table below. (The speed of sound in the air columns is 345 m/s.)

<table>
<thead>
<tr>
<th>Bottle</th>
<th>Length of Column (m)</th>
<th>Wavelength (m)</th>
<th>Frequency (Hz)</th>
<th>Speed (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.060</td>
<td>0.24</td>
<td>1434.5</td>
<td>345</td>
</tr>
<tr>
<td>B</td>
<td>0.122</td>
<td>0.487</td>
<td>708</td>
<td>345</td>
</tr>
<tr>
<td>C</td>
<td>0.16</td>
<td>0.640</td>
<td>539</td>
<td>345</td>
</tr>
<tr>
<td>D</td>
<td>0.200</td>
<td>0.8</td>
<td>431</td>
<td>345</td>
</tr>
</tbody>
</table>

\[ L = \frac{1}{4} \lambda \]
\[ f = \frac{\sqrt{\frac{v}{L}}}{4L} \]

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