\[ e = \frac{c^2}{\sqrt{3}} \]

88. Analyze and Conclude A ball on a light string moves in a vertical circle. Analyze and describe the motion of this system. Be sure to consider the effects of gravity and tension. Is this system in uniform circular motion? Explain your answer.

It is not uniform circular motion.
Gravity increases the speed of the ball when it moves downward and reduces the speed when it is moving upward. Therefore, the centripetal acceleration needed to keep it moving in a circle will be larger at the bottom and smaller at the top of the circle. At the top, tension and gravity are in the same direction, so the tension needed will be even smaller. At the bottom, gravity is outward while the tension is inward. Thus, the tension exerted by the string must be even larger.

Writing in Physics

89. Roller Coasters If you take a look at vertical loops on roller coasters, you will notice that most of them are not circular in shape. Research why this is so and explain the physics behind this decision by the coaster engineers.

Answers will vary. Since \( F_c = \frac{mv^2}{r} \), as \( v \) decreases due to gravity when going uphill, \( r \) is reduced to keep the force constant.

90. Many amusement-park rides utilize centripetal acceleration to create thrills for the park's customers. Choose two rides other than roller coasters that involve circular motion and explain how the physics of circular motion creates the sensations for the riders.
Chapter 6 continued

begins its ascent. What must the velocity of the balloon be for Shifty to easily catch the bag?

\[
\Delta x = v_{x1}t, \ \text{so} \ \Delta x = x - v_{x1}t
\]

\[
\Delta y_{bag} = v_{y1}t - \frac{1}{2}gt^2, \ \text{but} \ v_{y1} = 0
\]

\[
\text{so} \ \Delta y_{bag} = -\frac{1}{2}gt^2
\]

\[
v_{balloon} = \frac{\Delta y_{balloon}}{t}
\]

\[
= 60.0 \text{ m} - \frac{1}{2}gt^2
\]

\[
= 60.0 \text{ m} + \frac{1}{2}gt^2
\]

\[
= 60.0 \text{ m} + \frac{1}{2}gt^2
\]

\[
= \frac{(60.0 \text{ m})v_{x1} + gt}{x}
\]

\[
= \frac{(60.0 \text{ m})(7.3 \text{ m/s})}{(20.0 \text{ m})} + \frac{(-9.8 \text{ m/s})(20.0 \text{ m})}{60.0 \text{ m}}
\]

\[
= 8.5 \text{ m/s}
\]

Thinking Critically

page 168

S3. Apply Concepts. Consider a roller-coaster loop like the one in Figure 6-18. Are the cars traveling through the loop in uniform circular motion? Explain.

![Figure 6-18](image)

The vertical gravitational force changes the speed of the cars, so the motion is not uniform circular motion.

S4. Use the Numbers A 3-point jump shot is released 2.2 m above the ground and 6.02 m from the basket. The basket is 3.05 m above the floor. For launch angles of 30° and 60°, find the speed the ball needs to be thrown to make the basket.

S6. Analyze. Albert Einstein showed that the rate you learned for the addition of velocities does not work for objects moving near the speed of light. For example, if a rocket moving at velocity \( v_r \) releases a missile that has velocity \( v_k \) relative to the rocket, then the velocity of the missile relative to an observer that is at rest is given by \( v = (v_r - v_k)/(1 - v_rv_k/c^2) \), where \( c \) is the speed of light, 3.00 \times 10^8 \text{ m/s}. This formula gives the correct values for objects moving at slow speeds as well. Suppose a rocket moving at 1 km/s shoots a laser beam out in front of it. What speed would an unmarking observer find for the laser light? Suppose that a rocket moves at a speed \( c/2 \), half the speed of light, and shoots a missile forward at a speed of \( c/2 \) relative to the rocket. How fast would the missile be moving relative to a fixed observer?
Chapter 6 continued

78. A 1.13-kg ball is swung vertically from a 0.50-m cord in uniform circular motion at a speed of 2.4 m/s. What is the tension in the cord at the bottom of the ball’s motion?

\[ F_T = F_U + F_e = mg + \frac{mv^2}{r} = (1.13 \text{ kg})(9.80 \text{ m/s}^2) + (1.13 \text{ kg})(2.4 \text{ m/s})^2 = 24 \text{ N} \]

79. Banked Roads Curves on roads often are banked to help prevent cars from slipping off the road. If the posted speed limit for a particular curve of radius 36.0 m is 15.7 m/s (35 mph), at what angle should the road be banked so that cars can stay on a circular path even if there were no friction between the road and the tires? If the speed limit was increased to 26.1 m/s (45 mph), at what angle should the road be banked?

Level 3

80. The 1.45-kg ball in Figure 6-17 is suspended from a 0.80-m string and swung in a horizontal circle at a constant speed such that the string makes an angle of 14.0° with the vertical.

a. What is the tension in the string?

\[ F_T \cos \theta = mg \]
\[ \text{so } F_T = \frac{mg}{\cos \theta} = \frac{(1.45 \text{ kg})(9.80 \text{ m/s}^2)}{\cos 14.0°} = 14.6 \text{ N} \]

b. What is the speed of the ball?

\[ F_T \sin \theta = \frac{mv^2}{r} = F_T \cos \theta = \frac{mg}{\cos \theta} \]
\[ \text{so } F_T \sin \theta = \frac{mv^2}{r} = \frac{mg}{\cos \theta} \]
\[ \text{or } \tan \theta = \frac{v^2}{rg} \]

Chapter 6 continued

81. A baseball is hit directly in line with an outfielder at an angle of 35.0° above the horizontal with an initial velocity of 22.0 m/s. The outfielder starts running as soon as the ball is hit at a constant velocity of 2.5 m/s and barely catches the ball. Assuming such that the ball is caught at the same height in which it was hit, what was the initial separation between the hitter and outfielder? Time: There are two possible answers.

\[ \Delta x = v_x t \pm v_y f = ft(v_x \pm v_y) \]

To get t,
\[ y = v_y t - \frac{1}{2} gt^2, y = 0 \]
\[ \text{so } v_y t - \frac{1}{2} gt^2, t = 0 \text{ or } v_y t = \frac{1}{2} gt^2 \]
\[ t = \frac{2v_y}{g} = \frac{2v_y \sin \theta}{g} \]
\[ \text{so } \Delta x = \frac{2v_y \sin \theta}{g} (v_x \pm v_y) \]
\[ = \frac{2v_y \sin \theta}{g} (v_y \cos \theta \pm v_y) \]
\[ = \frac{(2)(22.0 \text{ m/s})\sin 35.0°}{9.80 \text{ m/s}^2} \]
\[ = \frac{(22.0 \text{ m/s}) \sin 35.0°}{2.5 \text{ m/s}} \]
\[ = 53 \text{ m or } 4.0 \times 10^3 \text{ m} \]

82. A Javelin Throw You are serving as a technical consultant for a locally produced cartoon. In one episode, two criminals, Shifty and Lefty, have stolen some jewels. Lefty has the jewels when the police start to chase him, and he runs to the top of a 60.0-m tall building in his attempt to escape. Meanwhile, Shifty runs to the convenient hot-air balloon 20.0 m from the base of the building and retrieves it, so it begins to rise at a constant speed. Lefty tosses the bag of jewels horizontally with a speed of 7.3 m/s just as the balloon...
Level 2

70. Crossing a River You row a boat, such as the one in Figure 6-16, perpendicular to the shore of a river that flows at 3.0 m/s. The velocity of your boat is 4.0 m/s relative to the water.

\[ v = \tan^{-1}\left(\frac{v_{ws}}{v_{bw}}\right) \]
\[ v_{ws} = \frac{v_{bw}}{\cos\theta} \]
\[ v_{bw} = \frac{v_{ws}}{\cos\theta} \]
\[ \theta = \tan^{-1}\left(\frac{v_{ws}}{v_{bw}}\right) \]
\[ \theta = \tan^{-1}\left(\frac{15 \text{ m/s}}{6.5 \text{ m/s}}\right) \]
\[ \theta = 67^\circ \text{ from the horizon toward the west} \]

Level 3

72. Boating You are boating on a river that flows toward the east. Because of your knowledge of physics, you head your boat 53\(^\circ\) west of north and have a velocity of 6.0 m/s due north relative to the shore.

a. What is the velocity of the current?

\[ v = \tan^{-1}\left(\frac{v_{ws}}{v_{bw}}\right) \]
\[ v_{ws} = \frac{v_{bw}}{\cos\theta} \]
\[ v_{bw} = \frac{v_{ws}}{\cos\theta} \]
\[ v_{bw} = 8.0 \text{ m/s east} \]

b. What is the speed of your boat relative to the water?

\[ v = \sqrt{v_{bw}^2 + (v_{ws})^2} \]
\[ v = \sqrt{(8.0 \text{ m/s})^2 + (3.0 \text{ m/s})^2} \]
\[ v = 8.5 \text{ m/s} \]

73. Air Travel You are piloting a small plane, and you want to reach an airport 450 km due south in 3.0 h. A wind is blowing from the west at 50.0 km/h. What heading and airspeed should you choose to reach your destination in time?

\[ v = \frac{d}{t} \]
\[ v = \frac{450 \text{ km}}{3.0 \text{ h}} \]
\[ v = 150 \text{ km/h} \]

\[ v = \sqrt{v_{bw}^2 + (v_{ws})^2} \]
\[ v = \sqrt{(150 \text{ km/h})^2 + (50.0 \text{ km/h})^2} \]
\[ v = 16 \text{ m/s} \]

\[ \theta = \tan^{-1}\left(\frac{v_{ws}}{v_{bw}}\right) \]
\[ \theta = \tan^{-1}\left(\frac{50.0 \text{ km/h}}{150 \text{ km/h}}\right) \]
\[ \theta = 18^\circ \text{ west of south} \]

Mixed Review
Chapter 6 continued

62. Hammer Throw An athlete whirls a 7.06-kg hammer 1.8 m from the axis of rotation in a horizontal circle, as shown in Figure 6-14. If the hammer makes one revolution in 1.0 s, what is the centripetal acceleration of the hammer? What is the tension in the chain?

\[ a_c = \frac{v^2}{r} \]

\[ (4 \pi \text{ rad}/1.8 \text{ m}) = 2.71 \text{ m/s}^2 \]

\[ a_c = \frac{(4 \pi \text{ rad})(1.0 \text{ s})}{1.8 \text{ m}} = -1.22 \text{ m/s}^2 \]

\[ r = 15.0 \text{ cm} \]

\[ a_c = \frac{v^2}{r} \]

\[ (4 \pi \text{ rad})(0.150 \text{ m}) = 1.94 \text{ m/s}^2 \]

\[ a_c = \frac{v^2}{r} = \frac{(4 \pi \text{ rad})(0.150 \text{ m})}{1.50 \text{ s}} = -1.53 \text{ m/s}^2 \]

\[ c. \text{ What force accelerates the coin?} \]

Frictional force between coin and record.

\[ d. \text{ At which of the three radii in part b would the coin be most likely to fly off the turntable? Why?} \]

15.0 cm, the largest radius; the frictional force needed to hold it is the greatest.

64. A rotating rod that is 15.3 cm long is spun with its axis through one end of the rod so that the other end of the rod has a speed of 200 m/s (450 mph).

\[ a_c = \frac{v^2}{r} \]

\[ (200 \text{ m/s}) = \frac{v^2}{0.153 \text{ m}} \]

\[ v = \sqrt{2.64 \times 10^4 \text{ m}^2/\text{s}^2} \]

\[ b. \text{ If you were to attach a 1.0-g object to the end of the rod, what force would be needed to hold it on the rod?} \]

\[ F_c = ma_c \]

\[ (0.0010 \text{ kg})(2.64 \times 10^4 \text{ m}^2/\text{s}^2) \]

\[ = 2.6 \times 10^{-5} \text{ N} \]

65. Friction provides the forces needed for a car to travel around a flat, circular race track. What is the maximum speed at which a car can safely travel if the radius of the track is 80.0 m and the coefficient of friction is 0.40?

\[ F_c = F_f = \mu F_n = \mu mg \]

But \( F_c = \frac{mv^2}{r} \), thus \( mv^2 = \mu mg \).

Chapter 6 continued

The mass of the car divides out to give

\[ v^2 = \mu gr, \text{ so} \]

\[ v = \sqrt{\mu gr} \]

\[ = \sqrt{(0.40)(9.80 \text{ m/s}^2)(80.0 \text{ m})} \]

\[ = 18 \text{ m/s} \]

Level 3

66. A carnival clown rides a motorcycle down a ramp and around a vertical loop. If the loop has a radius of 18 m, what is the slowest speed the rider can have at the top of the loop to avoid falling? Hint: At this slowest speed, the track exerts no force on the motorcycle at the top of the loop.

\[ F_c = ma_c = F_g = mg \text{, so} \]

\[ a_c = g \]

\[ r = \frac{v^2}{g} \]

\[ \frac{v^2}{g} = 18 \text{ m/s} \]

6.3 Relative Velocity

Level 1

68. Navigating an Airplane An airplane flies at 200.0 km/h relative to the air. What is the velocity of the plane relative to the ground if it flies during the following wind conditions?

a. A 50.0-km/h tailwind

Tailwind is in the same direction as the airplane

\[ 200.0 \text{ km/h} + 50.0 \text{ km/h} = 250.0 \text{ km/h} \]

b. A 50.0-km/h headwind

Head wind is in the opposite direction of the airplane

\[ 200.0 \text{ km/h} - 50.0 \text{ km/h} = 150.0 \text{ km/h} \]

68. Odina and LaToya are sitting by a river and decide to have a race. Odina will run down the shore to a dock, 1.5 km away, then turn around and run back. LaToya will also race to the dock and back, but she will row a boat in the river, which has a current of 2.0 m/s. If Odina's running speed is equal to LaToya's rowing speed in still water, which is 4.0 m/s, who will win the race? Assume that they both turn instantaneously.

\[ x = vt, \text{ so} \]

\[ t = \frac{x}{v} \]

for Odina,

\[ t = \frac{3.0 \times 10^3 \text{ m}}{4.0 \text{ m/s}} \]

\[ = 7.5 \times 10^2 \text{ s} \]

For LaToya (assume against current on the way to the dock),

\[ t = \frac{x_1}{v_1} + \frac{x_2}{v_2} \]
49. Car Racing: The curves on a race track are banked to make it easier for cars to go around the curves at high speeds. Draw a free-body diagram of a car on a banked curve. From the motion diagram, find the direction of the acceleration.

Front View

Top View

The acceleration is directed toward the center of the track.

a. What event takes the force in the direction of the acceleration?

The component of the normal force acting toward the center of the curve, and depending on the car's speed, the component of the friction force acting toward the center, both contribute to the net force in the direction of acceleration.

b. Can you have such a force without friction?

Yes, the centripetal acceleration need only be due to the normal force.

50. Driving on the Highway: Explain why it is that when you pass a car going in the same direction as you on the freeway, it takes a longer time than when you pass a car going in the opposite direction.

The relative speed of two cars going in the same direction is less than the relative speed of two cars going in the opposite direction. Passing with the lesser relative speed will take longer.

51. You accidentally throw your car keys horizontally at 8.0 m/s from a cliff 0.932 m high. How far from the base of the cliff should you look for the keys?

\[ y = v_y t - \frac{1}{2} gt^2 \]

Since initial vertical velocity is zero,

\[ t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{-1.292}{-9.80 \text{ m/s}^2}} \]

\[ t = 3.8 \text{ s} \]

\[ x = v_x t = (8.0 \text{ m/s})(3.6) = 28.8 \text{ m} \]

\[ x = 29 \text{ m} \]

52. The toy car in Figure 6.12 runs off the edge of a table that is 1.225 m high. The car lands 0.400 m from the base of the table.

\[ y = v_y t - \frac{1}{2} gt^2 \]

Since initial vertical velocity is zero,

\[ t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{1.225}{9.80 \text{ m/s}^2}} \]

\[ t = 0.500 \text{ s} \]

53. A dart player throws a dart horizontally at 12.4 m/s. The dart hits the board 0.32 m below the height from which it was thrown. How far from the base of the cliff does the dart hit the board?

\[ y = v_y t - \frac{1}{2} gt^2 \]

and because initial velocity is zero,

\[ t = \sqrt{\frac{2y}{g}} \]

54. The two baseballs in Figure 6.13 were hit with the same speed, 25 m/s. Draw separate graphs of \( y \) versus \( t \) and \( x \) versus \( t \) for each ball.

55. Swimming: You took a running leap off a high diving platform. You were running at 2.8 m/s and hit the water 2.6 s later. How high was the platform, and how far from the edge of the platform did you hit the water? Ignore air resistance.

\[ y = v_y t - \frac{1}{2} gt^2 \]

\[ V = (2.6 \text{ s}) \left[ \frac{1}{2} \left(9.80 \text{ m/s}^2\right) (2.6 \text{ s}) \right]^2 \]

\[ = 0 \text{ m} \]

\[ x = (2.8 \text{ m/s})(2.6 \text{ s}) - 7.3 \text{ m} \]

56. Archery: An arrow is shot at 30.0° above the horizontal. Its velocity is 49 m/s, and it hits the target.

a. What is the maximum height the arrow will attain?

\[ v_y^2 = v_{y0}^2 - 2gd \]

At the high point \( v_y = 0 \), so

\[ d = \frac{(v_{y0})^2}{2g} \]

\[ = \frac{[49 \text{ m/s} \times \sin 30.0°]^2}{2(9.80 \text{ m/s}^2)} \]

\[ = 21 \text{ m} \]

b. The target is at the height from which the arrow was shot. How far away is it?

\[ y = v_{y0} t - \frac{1}{2} gt^2 \]

but the arrow lands at the same height, so

\[ y = 0 \text{ and } 0 = v_{y0} t - \frac{1}{2} gt \]

so \( t = 0 \) or

\[ t = \frac{2v_{y0}}{g} \]

\[ = \frac{2(49 \text{ m/s}) \sin 30.0°}{9.80 \text{ m/s}^2} \]

\[ = 5.0 \text{ s} \]
57. Hitting a Home Run A pitched ball is hit by a batter at a 45° angle and just clears the outfield fence, 98 m away. If the fence is at the same height as the pitcher, find the velocity of the ball when it left the bat, ignoring air resistance.

The components of the initial velocity are \( v_x = v_i \cos \delta \) and \( v_y = v_i \sin \delta \).

Now \( x = v_{yt} t \), so \( t = \frac{x}{v_y} \), thus \( v_y \) = \( \sqrt{\frac{2y}{t}} \) for \( y = \frac{1}{2} gt^2 \), but \( y = 0 \), so

\[ t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2 \times 98}{9.8}} = 4.6 \text{ s} \]

\[ x = \frac{1}{2} \times 9.8 \times (4.6)^2 = 115 \text{ m} \]

b. What is the horizontal distance between the plane and the victims when the box is dropped?

\[ x = \frac{1}{2} \times 9.8 \times (4.6)^2 = 115 \text{ m} \]

\[ x = \frac{1}{2} \times 9.8 \times (4.6)^2 = 115 \text{ m} \]

58. Diving Divers in Acapulco dive from a cliff that is 61 m high. If the rocks below the cliff extend outward for 23 m, what is the minimum horizontal velocity a diver must have to clear the rocks?

\[ y = v_{yt} t - \frac{1}{2} gt^2 \]

and since \( v_y = 0 \),

\[ t = \frac{-v_{yt}}{g} \]

\[ v_x = \frac{v_{xt}}{t} = \frac{23}{3.53} = 6.5 \text{ m/s} \]

60. Jump Shot A basketball player is trying to make a half-court jump shot and releases the ball at the height of the basket. Assuming that the ball is launched at 31.0°, 14.0 m from the basket, what speed must the player give the ball?

The components of the initial velocity are \( v_x = v_i \cos \delta \) and \( v_y = v_i \sin \delta \).

Now \( x = v_{xt} t = (v_i \cos \delta) t \), so
37. To obtain uniform circular motion, how must the net force depend on the speed of the moving object? [6.2]

Circular motion results when the direction of the force is constantly perpendicular to the instantaneous velocity of the object.

38. If you whirl a yo-yo about your head in a horizontal circle, in what direction must a force act on the yo-yo? What exerts the force? [6.2]

The force is along the string toward the center of the circle that the yo-yo follows. The string exerts the force.

39. Why is it that a car traveling in the opposite direction as the car in which you are riding on the freeway often looks like it is moving faster than the speed limit? [6.3]

The magnitude of the relative velocity of that car to your car can be found by adding the magnitudes of the two cars' velocities together. Since each car is probably moving at close to the speed limit, the resulting relative velocity will be larger than the posted speed limit.

40. Baseball A batter hits a pop-up straight up over home plate at an initial velocity of 20 m/s. The ball is caught by the catcher at the same height that it was hit. At what velocity does the ball land in the catcher's mitt? Neglect air resistance.

- 20 m/s, where the negative sign indicates down

41. Fastball In baseball, a fastball takes about 0.3 s to reach the plate. Assuming that such a pitch is thrown horizontally, compare the distance the ball falls in the first 0.3 s with the distance it falls in the second 0.3 s.

Because of the acceleration due to gravity, the baseball falls a greater distance during the second 0.3 s than during the first 0.3 s.

42. You throw a rock horizontally. In a second horizontal throw, you throw the rock harder and give it even more speed.

a. How will the time it takes the rock to hit the ground be affected? Ignore air resistance.

The time does not change—the time it takes to hit the ground depends only on vertical velocities and acceleration.

b. How will the increased speed affect the distance from where the rock left your hand to where the rock hits the ground?

A higher horizontal speed produces a longer horizontal distance.

43. Field Biology A zoologist standing on a hill aims a tranquilizer gun at a monkey hanging from a distant tree branch. The barrel of the gun is horizontal. Just as the zoologist pulls the trigger, the monkey lets go and begins to fall. Will the dart hit the monkey? Ignore air resistance.

Yes, in fact, the monkey would be safe if it did not let go of the branch. The vertical acceleration of the dart is the same as that of the monkey. Therefore, the dart is at the same vertical height when it reaches the monkey.

44. Football A quarterback throws a football at 24 m/s at a 45° angle. If it takes the ball 3.0 s to reach the top of its path and the ball is caught at the same height at which it is thrown, how long is it in the air? Ignore air resistance.

6.0 s: 3.0 s up and 3.0 s down

45. Track and Field You are working on improving your performance in the long jump and believe that the information in this chapter can help. Does the height that you reach make any difference to your jump? What influences the length of your jump?

Both speed and angle of launch matter, so height does make a difference.

Maximum range is achieved when the resultant velocity has equal vertical and horizontal components—in other words, a launch angle of 45°. For this reason, height and speed affect the range.

46. Imagine that you are sitting in a car towing a ball straight up into the air.

a. If the car is moving at a constant velocity, will the ball land in front of, behind, or in your hand?

The ball will land in your hand because you, the ball, and the car all are moving forward with the same speed.

b. If the car rounds a curve at a constant speed, where will the ball land?

The ball will land beside you, toward the outside of the curve. A top view would show the ball moving straight while you and the car moved out from under the ball.

47. You swing one yo-yo around your head in a horizontal circle. Then you swing another yo-yo with twice the mass of the first one, but you don't change the length of the string or the period. How do the tensions in the strings differ?

The tension in the string is doubled since $T = m_2 a_2$.
30. Relative Velocity An airplane has a speed of 285 km/h relative to the air. There is a wind blowing at 95 km/h at 30.0° north of east relative to Earth. In which direction should the plane head to land at an airport due north of its present location? What is the plane's speed relative to the ground?

To travel north, the east components must be equal and opposite.

\[
\cos \theta_w = \frac{V_{ew}}{V_{ew}} \quad \text{so} \quad \theta_w = \cos^{-1} \left( \frac{V_{ew}}{V_{ew}} \right) = \cos^{-1} \left( \frac{95 \text{ km/h}}{285 \text{ km/h}} \cos 30.0° \right) = 73° \text{ north of west}
\]

\[
V_{we} = V_{ew} + V_{wn} = V_e \sin \theta_w + V_w \sin \theta_w = (285 \text{ km/h}) (\sin 107°) + (95 \text{ km/h}) (\sin 30.0°) = 320 \text{ km/h}
\]

31. Critical Thinking You are piloting a boat across a fast-moving river. You want to reach a pier directly opposite your starting point. Describe how you would navigate the boat in terms of the components of your velocity relative to the water.

You should choose the component of your velocity along the direction of the river to be equal and opposite to the velocity of the river.

Chapter Assessment
Concept Mapping

32. Use the following terms to complete the concept map below: constant speed, constant acceleration, horizontal part of projectile motion, constant velocity, relative velocity motion, uniform circular motion.

```
Categories of Motion

constant acceleration
constant velocity
constant speed
vertical part of projectile motion
relative velocity motion
horizontal part of projectile motion
uniform circular motion
```

Mastering Concepts

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33. Consider the trajectory of the cannonball shown in Figure 6-11. (6.1)

Up is positive, down is negative.

a. Where is the magnitude of the vertical velocity component largest?
   The greatest vertical velocity occurs at point A.

b. Where is the magnitude of the horizontal velocity component largest?
   Neglecting air resistance, the horizontal velocity at all points is the same. Horizontal velocity is constant and independent of vertical velocity.

c. Where is the vertical velocity smallest?
   The least vertical velocity occurs at point E.

d. Where is the magnitude of the acceleration smallest?
   The magnitude of the acceleration is the same everywhere.

34. A student is playing with a radio-controlled race car on the balcony of a sixth-floor apartment. An accidental turn sends the car through the railing and over the edge of the balcony. Does the time it takes the car to fall depend upon the speed it had when it left the balcony? (6.1)

No, the horizontal component of motion does not affect the vertical component.

35. An airplane pilot flying at constant velocity and altitude drops a heavy crate. Ignoring air resistance, where will the plane be relative to the crate when the crate hits the ground? Draw the path of the crate as seen by an observer on the ground. (6.1)

The plane will be directly over the crate when the crate hits the ground. Both have the same horizontal velocity. The crate will look like it is moving horizontally while falling vertically to an observer on the ground.

36. Can you go around a curve with the following accelerations? Explain.

a. zero acceleration
   No, going around a curve causes a change in direction of velocity. Thus, the acceleration cannot be zero.

b. constant acceleration (6.2)
   No, the magnitude of the acceleration may be constant, but the direction of the acceleration changes.
Practice Problems

6.3 Relative Velocity pages 157–159

22. You are riding in a bus moving slowly through heavy traffic at 2.0 m/s. You hurry to the front of the bus at 4.0 m/s relative to the bus. What is your speed relative to the street?

\[
V_{\text{bus}} = V_{\text{rel}} + V_{\text{rel}} = 2.0 \text{ m/s} + 4.0 \text{ m/s} = 6.0 \text{ m/s relative to street}
\]

23. You are pulling a toy wagon through the neighborhood at a speed of 0.75 m/s. A caterpillar is crawling toward the rear of the wagon at a rate of 2.0 cm/s. What is the caterpillar's velocity relative to the ground?

\[
V_{\text{cat}} = V_{\text{rel}} + V_{\text{rel}} = 0.75 \text{ m/s} - 0.02 \text{ m/s} = 0.73 \text{ m/s}
\]

24. A boat is rowed directly upstream at a speed of 2.5 m/s relative to the water. Viewers on the shore see that the boat is moving at only 0.5 m/s relative to the shore. What is the speed of the river? Is it moving with or against the boat?

\[
V_{\text{rel}} = V_{\text{rel}} + V_{\text{rel}} = 2.5 \text{ m/s} - 0.5 \text{ m/s} = 2.0 \text{ m/s against the boat}
\]

Section Review

6.3 Relative Velocity pages 157–159

26. Relative Velocity A fishing boat with a maximum speed of 3 m/s relative to the water is in a river that is flowing at 2 m/s. What is the maximum speed the boat can obtain relative to the shore? The minimum speed? Give the direction of the boat relative to the river's current, for the maximum speed and the minimum speed relative to the shore.

The maximum speed relative to the shore is when the boat moves at maximum speed in the same direction as the river's flow:

\[
V_{\text{rel}} = V_{\text{rel}} + V_{\text{rel}} = 3 \text{ m/s} + 2 \text{ m/s} = 5 \text{ m/s}
\]

The minimum speed relative to the shore is when the boat moves at maximum speed in the opposite direction of the river's flow with the same speed as the river:

\[
V_{\text{rel}} = V_{\text{rel}} + V_{\text{rel}} = 3 \text{ m/s} - 2 \text{ m/s} = 1 \text{ m/s}
\]

27. Relative Velocity of a Boat A powerboat heads due north at 13 m/s relative to the water across a river that flows due north at 5.0 m/s. What is the velocity (both magnitude and direction) of the motorboat relative to the shore?

\[
V_{\text{boat}} = V_{\text{rel}} + V_{\text{rel}} = 13 \text{ m/s} + 5 \text{ m/s} = 18 \text{ m/s}
\]

\[
\theta = \tan^{-1}\left(\frac{V_{\text{rel}}}{V_{\text{rel}}}ight) = \tan^{-1}\left(\frac{13 \text{ m/s}}{-5 \text{ m/s}}\right) = -72^\circ
\]
Chapter 6 continued

14. An airplane traveling at 201 m/s makes a turn. What is the smallest radius of the circular path (in km) that the pilot can make and keep the centripetal acceleration under 5.0 m/s²?

\[ a_c = \frac{v^2}{r}, \quad \text{so} \quad r = \frac{v^2}{a_c} = \frac{(201 \text{ m/s})^2}{5.0 \text{ m/s}^2} = 8.1 \text{ km} \]

15. A 45-kg merry-go-round worker stands on the ride's platform 6.3 m from the center. If her speed as she goes around the circle is 4.1 m/s, what is the force of friction necessary to keep her from falling off the platform?

\[ F_f = F_c = \frac{mv^2}{r} = \frac{(45 \text{ kg})(4.1 \text{ m/s})^2}{6.3 \text{ m}} = 120 \text{ N} \]

Section Review

6.2 Circular Motion

page 156

16. Uniform Circular Motion What is the direction of the force that acts on the clothes in the spin cycle of a washing machine? What exerts the force?

The force is toward the center of the tub. The walls of the tub exert the force on the clothes. Of course, the whole point is that some of the water in the clothes goes out through holes in the wall of the tub rather than moving toward the center.

17. Free-Body Diagram You are sitting in the backseat of a car going around a curve to the right. Sketch a free-body diagram and free-body diagrams to answer the following questions.

\[ \sum F_{net} = F_c \]

\[ \vec{F}_{net} + \vec{F}_c = \vec{F}_s \]

a. What is the direction of your acceleration?

Your body is accelerated to the right.

b. What is the direction of the net force that is acting on you?

The net force acting on your body is to the right.

c. What exerts this force?

The force is exerted by the car's seat.

18. Centripetal Force If a 40.0-g stone is whirled horizontally on the end of a 0.60-m string at a speed of 2.2 m/s, what is the tension in the string?

\[ F_T = \frac{mv^2}{r} = \frac{(0.0400 \text{ kg})(2.2 \text{ m/s})^2}{0.60 \text{ m}} = 0.32 \text{ N} \]

19. Centripetal Acceleration A newspaper article states that when turning a corner, a driver must be careful to balance the centripetal and centrifugal forces to keep from skidding. Write a letter to the editor that critiques this article.

The letter should state that there is an acceleration because the direction of the velocity is changing; therefore, there must be a net force in the direction of the center of the circle. The road supplies that force and the friction between the road and the tires allows the force to be exerted on the tires. The car's seat exerts the force on the driver that accelerates him or her toward the center of the circle. The note also should make it clear that centrifugal force is not a real force.

20. Centripetal Force A bowling ball has a mass of 7.3 kg. If you move it around a circle with a radius of 0.75 m at a speed of 2.5 m/s, what force would you have to exert on it?

\[ F_{net} = ma_c = \frac{mv^2}{r} \]
Chapter 6 continued

Distance:
\[ x = v_x \cos \theta \cdot t \]
\[ = (27.0 \text{ m/s})(\cos 60.0^\circ)(4.77 \text{ s}) \]
- 64.4 m

Maximum height:
\[ y = v_y \sin \theta \cdot t - \frac{1}{2}gt^2 \]
\[ = (27.0 \text{ m/s})(\sin 60.0^\circ)(2.38 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(2.38 \text{ s})^2 \]
\[ = 27.9 \text{ m} \]

6. A rock is thrown from a 50.0-m-high cliff with an initial velocity of 7.0 m/s at an angle of 35.0° above the horizontal. Find the velocity vector for when it hits the ground below.

\[ v_x = v_x \cos \theta \]
\[ v_y = v_y \sin \theta + gt \]
\[ = v_y \sin \theta + g \sqrt{\frac{2y}{g}} \]
\[ = v_y \sin \theta - \sqrt{2yg} \]
\[ v = \sqrt{v_x^2 + v_y^2} \]
\[ = \sqrt{(v_x \cos \theta)^2 + (v_y \sin \theta + \sqrt{2yg})^2} \]
\[ = \sqrt{((7.0 \text{ m/s}) \cos 53.0^\circ)^2 + ((7.0 \text{ m/s}) \sin 53.0^\circ + \sqrt{2(9.80 \text{ m/s}^2)(25.0 \text{ m})})^2} \]
\[ = 37 \text{ m/s} \]
\[ \theta = \tan^{-1} \left( \frac{v_y}{v_x} \right) \]
\[ = \tan^{-1} \left( \frac{v_y \sin \theta + \sqrt{2yg}}{v_x \cos \theta} \right) \]
\[ = \tan^{-1} \left( \frac{(7.0 \text{ m/s}) \sin 53.0^\circ + \sqrt{(9.80 \text{ m/s}^2)(25.0 \text{ m})}}{(7.0 \text{ m/s}) \cos 53.0^\circ} \right) \]
\[ = 83^\circ \text{ from horizontal} \]

Section Review

6.1 Projectile Motion pages 147–152

7. Projectile Motion Two baseballs are pitched horizontally from the same height, but at different speeds. The faster ball crosses home plate within the strike zone, but the slower ball is below the batter's knees. Why does the faster ball not fall as far as the slower one?

Chapter 6 continued

The faster ball is in the air a shorter time, and thus gains a smaller vertical velocity.

8. Free-Body Diagram An ice cube slides without friction across a table at a constant velocity. It slides off the table and lands on the floor. Draw free-body and motion diagrams of the ice cube at two points in the air:

Free-Body Diagrams

- On the table: \( F_x = 0 \), \( F_y = 0 \)
- In the air: \( F_x = 0 \), \( F_y = -mg \)

Motion Diagrams

- On the table: \( v_x = 0 \), \( v_y = 0 \)
- In the air: \( v_x = v_{0x} \), \( v_y = v_{0y} - gt \)

8. Projectile Motion A softball is tossed into the air at an angle of 50.0° with the vertical at an initial velocity of 11.0 m/s. What is its maximum height?

\[ v^2 = v_{0y}^2 + 2a(d_y - d_i) \]

At maximum height \( v_y = 0 \), so

\[ d_y - d_i = \frac{v_{0y}^2}{2g} \]

\[ = \frac{(11.0 \text{ m/s}) \cos 50.0^\circ)^2}{2 \times (9.80 \text{ m/s}^2)} \]

\[ = 2.55 \text{ m} \]

10. Projectile Motion A tennis ball is thrown out a window 28 m above the ground at an initial velocity of 15.0 m/s and 20.0° below the horizontal. How far does the ball move horizontally before it hits the ground?

\[ x = v_{0x}t \text{ but need to find } t \]

First, determine \( v_{0y} \):

\[ v_{0y}^2 = v_{0y}^2 + 2gy \]

\[ v_{0y} = \sqrt{v_{0y}^2 + 2gy} \]

\[ = \sqrt{((15.0 \text{ m/s}) \sin 20.0^\circ)^2 + 2(9.80 \text{ m/s}^2)(28 \text{ m})} \]

\[ = 24.0 \text{ m/s} \]

Now use \( v_{0y} = v_{0y} + gt \) to find \( t \):

\[ t = \frac{v_{0y} - v_{0y}}{g} \]

\[ = \frac{v_{0y} - v_{0y}}{g} \]
Motion in Two Dimensions

Practice Problems

6.1 Projectile Motion

1. A stone is thrown horizontally at a speed of 5.0 m/s from the top of a cliff that is 78.4 m high.
   a. How long does it take the stone to reach the bottom of the cliff?

   Since \( v_y = 0 \), \( y = v_{y}t - \frac{1}{2}gt^2 \)
   becomes \( y = \frac{1}{2}gt^2 \)
   or \( t = \sqrt{\frac{2y}{g}} \)
   \( = \sqrt{\frac{2(78.4)}{9.8}} \)
   \( = 4.00 \text{ s} \)

   b. How far from the base of the cliff does the stone hit the ground?

   \( x = v_{x}t = (5.0 \text{ m/s})(4.00 \text{ s}) \)
   \( = 20 \times 10^1 \text{ m} \)

   c. What are the horizontal and vertical components of the stone's velocity just before it hits the ground?

   \( v_x = 5.0 \text{ m/s} \). This is the same as the initial horizontal speed because the acceleration of gravity influences only the vertical motion. For the vertical component, use \( v_y = v_{y0} + gt \) with \( v_y = v_{y0} \) and \( v_y \), the initial vertical component of velocity, zero.

   At \( t = 4.00 \text{ s} \)
   \( v_y = gt \)
   \( = (9.8 \text{ m/s}^2)(4.00 \text{ s}) \)
   \( = 39.2 \text{ m/s} \)

2. Lucy and her friend are working at an assembly plant making wooden toy giraffes. At the end of the line, the giraffes go horizontally off the edge of the conveyor belt and fall into a box below. If the box is 0.6 m below the level of the conveyor belt and 0.4 m away from it, what must be the horizontal velocity of giraffes as they leave the conveyor belt?

   \( x = v_{x}t = v_{x}\frac{\sqrt{2y}}{g} \)
   so \( v_{x} = \frac{x}{\sqrt{\frac{2y}{g}}} \)
   \( = \frac{0.4 \text{ m}}{\sqrt{\frac{2(0.6)}{9.8}} \text{ m}^2/\text{s}^2}} \)
   \( = 1 \text{ m/s} \)

3. You are visiting a friend from elementary school who now lives in a small town. One local amusement is the ice-cream parlor, where Stan, the short-order cook, slides his completed ice-cream sundae down the counter at a constant speed of 2.0 m/s to the servers. (The counter is kept very well polished for this purpose.) If the servers catch the sundae 7.0 cm from the edge of the counter, how far do they fall from the edge of the counter to the point at which the servers catch them?

   \( x = v_{x}t \)
   \( t = \frac{x}{v_{x}} \)
   \( y = \frac{1}{2}gt^2 \)
   \( = \frac{1}{2}g\frac{x^2}{v_{x}^2} \)
   \( = \frac{1}{2}(9.8 \text{ m/s}^2)(0.070 \text{ m})^2 \)
   \( = 0.0060 \text{ m or } 60.0 \text{ cm} \)

4. A player kicks a football from ground level with an initial velocity of 27.0 m/s, 30° above the horizontal, as shown in Figure 6-4. Find each of the following. Assume that air resistance is negligible.

   a. the ball's hang time

   \( v_y = v_{y0} \sin \theta \)
   \( = 27.0 \text{ m/s} \sin 30.0° \)
   \( = 13.5 \text{ m/s} \)

   When it lands, \( y = v_{y}t - \frac{1}{2}gt^2 = 0. \)

   Therefore,

   \( t^2 = \frac{2v_{y}}{g} \)
   \( t = \frac{2v_{y}}{g} \)
   \( = \frac{2v_{y}}{g} \)
   \( = \frac{2v_{y} \sin \theta}{g} \)
   \( = \frac{2(27.0 \text{ m/s}) \sin 30.0°}{9.8 \text{ m/s}^2} \)
   \( = 2.76 \text{ s} \)

   b. the ball's maximum height

   Maximum height occurs at half the "hang time," or 1.38 s. Thus,

   \( y = v_{y}t - \frac{1}{2}gt^2 \)

   \( = v_{y} \sin \theta \cdot t - \frac{1}{2}gt^2 \)

   \( = (27.0 \text{ m/s}) \sin 30.0° (1.38 \text{ s}) - \frac{1}{2}(9.8 \text{ m/s}^2)(1.38 \text{ s})^2 \)
   \( = 9.30 \text{ m} \)

   a. the ball's range

   Distance:

   \( v_x = v_{x0} \cos \theta \)

   \( x = v_{x}t = (v_{x0} \cos \theta)(t) = (27.0 \text{ m/s}) \cos 30.0° (2.76 \text{ s}) \)
   \( = 64.5 \text{ m} \)

5. The player in Problem 4 then kicks the ball with the same speed, but at 60.0° from the horizontal. What is the ball's hang time, range, and maximum height?

   Following the method of Practice Problem 4,

   Hangtime:

   \( t = \frac{2v_{y} \sin \theta}{g} \)

   \( = \frac{2(27.0 \text{ m/s}) \sin 60.0°}{9.8 \text{ m/s}^2} \)
   \( = 4.77 \text{ s} \)